Journal of Materials and Engineering Structures

Research Paper

Two-dimensional fracture analysis of FGM under mechanical loading

Imène HEBBAR, Abdelkader BOULENOUAR, Yazid AIT FERHAT *

Laboratoire de Matériaux et Systèmes Réactifs, Université Djillali Liabes de Sidi-Bel-Abbès, BP 89, 22000, Algeria

ARTICLE INFO

Article history :

Received : 22 November 2019

Revised : 11 April 2020

Accepted : 15 April 2020

Keywords:

Displacement Extrapolation

Functionally Graded Material

Stress Intensity Factors

Mechanical Loading

ABSTRACT

This paper extends the concept to isotropic functionally graded materials and addresses fracture problems under mechanical loading. The mode-I and mixed mode stress intensity factors (SIFs) are determinate by combination between the finite element method and the displacement extrapolation technique (DET). The variation continue of the elastics properties is incorporated at the centroid of each finite element, using the Ansys Parametric Design Language (APDL). In this work, two examples are analysed to check for the robustness of the present approach, the FGM disk with a central inclined crack subjected to concentrated couple forces and the three-point bending specimen with crack parallel to material gradation. The numerical results obtained by present technique are discussed by comparison with other published results.

1 Introduction

The estimation of mixed mode stress intensity factors (SIFs) for fracture problems by means of the finite element method (FEM) is a technique widely employed to analyse non standard crack configurations. The evaluation methods can be categorized into two groups, those which use the energy release when the crack propagates and those based on field extrapolation around the crack tip [1, 2]. The first group includes the elemental crack extension, J-contour integration, the stiffness derivative method [3] and the energy domain integral formulation [4,5]. The main disadvantage of these approaches is that SIFs K_I and K_{II} , for mixed mode application, are very difficult to be separated between them. While the latter group which requires finer meshes to produce a better numerical representation of stress fields near the crack-tip. The singular elements are usually generated to facilitate the SIFs evaluation. For most commercial finite element codes, stress intensity factors are easily computed directly from nodal displacements, without resort to specialized post-processing routines, which are not always implemented in the commercial programs. In addition, the displacement techniques are still widely used

* *Corresponding author. Tel.: +213 781371879.*
E-mail address: aitferhatyazid@gmail.com

e-ISSN: 2170-127X,

RESEARCH REVIEW of Sciences and Technologies

because the continuous improvement of computers has reduced memory and computing time requirements. One of the most frequently used methods is displacement extrapolation (DET), which will be investigated in this study.

Functionally graded materials (FGMs) are one of the modern composites materials involving spatially varying volume fractions of constituent materials, thus providing a graded microstructure and macro properties. FGMs are used in many engineering applications, including aerospace, transportation, electronics, bio-medical, etc. For fracture problems, various techniques have been employed to determine the stress intensity factors (SIFs) in grades materials. Gu et al. [6] studied the crack-tip field of FGMs by a simplified method, using the equivalent domain integral (EDI) technique and Anlas et al. [7] evaluated the mode-I SIFs considering the modified integral method. The interaction integral method has been employed to determine the components of SIFs for isotropic and orthotropic homogeneous materials [8]. Dolbow and Gosz [9] investigated the interaction integral technique to extract the modes I and II SIFs in isotropic FGM materials. Gao et al. [10] presented 2D crack studies in non-homogeneous isotropic and linear elastic FGMs by means a boundary-domain integral formulation. Rao and Rahman [11] extended EFGM to determine the fracture parameters of isotropic FGM using two interaction integrals in terms nonhomogeneous and of homogenous auxiliary fields. Kim and Paulino [12] extended various finite elements based approaches for FGMs fracture analysis such as interaction integral, mixed-mode J-integral and modified crack closure methods. Dag and Yildirim [13] described the Jk-integral method for inclined cracks in graded materials subjected thermal loads. Yildirim et al. [14] analysed delamination behavior of orthotropic FGM coatings using the displacement correlation method. A modified form of the J-integral for axisymmetric FG cylinders under applied mechanical loads was proposed in [15] and was used to obtain reference SIFs from the FE analysis results. Benamara et al. [16] implemented the displacement extrapolation technique to study the fracture and the crack propagation problem in isotropic FGMs. Kim and Paulino [17] investigated the equivalent domain integral approach to calculate the mixed mode SIFs in FGMs, using the path-independent Jk*-integral. Recently, Ait Ferhat et al. [18] implemented the generalized displacement correlation (GDC) technique to determine the SIFs under mechanical and thermal fracture in isotropic FGMs.

The objective of the current study is to present a finite element numerical modeling of fracture problems for isotropic FGMs. Using the APDL code [19], the displacement extrapolation method is employed to determine numerically the SIFs under mode I and mixed mode loadings. In this work, the effect of material gradation and crack direction is analysed. Two different standard geometries are considered in this work, the FGM disk with a central inclined crack subjected to concentrated couple forces and the three-point bending specimen with crack parallel to material gradation. The results obtained in this study are compared with available solutions in literature.

The present paper is organized as follows: Section 2 presents the numerical computation of mixed mode SIFs. The two proposed examples are studied numerically in Section 3, considering a central inclined crack in isotropic FGM disk and a crack parallel to material gradation in three-point bending specimen. The paper closes in Section 4 with some comments and conclusions.

2 Evolution of stress intensity factors

The displacement extrapolation technique is one of the simplest and most frequently used methods [20-22]. For linear elastic solids and under mode-I loading, the displacement normal to crack plane, v is expressed by [7]:

$$v = K_I \frac{1+v}{4E}\sqrt{\frac{2r}{\pi}}\left[(2k+1)\,sin\frac{\theta}{2} - sin\frac{3\theta}{2}\right] + \frac{A_1(1+v)r}{E}(k-3)\,sin\,\theta + \frac{A_2(1+v)r^{\frac{3}{2}}}{E}\left[\frac{(2k-1)}{3}sin\frac{3\theta}{2} - sin\frac{\theta}{2}\right] + \cdots \tag{1}$$

Where K_I is the SIF under pure mode-I, E is the elasticity modulus, v is the Poisson's ratio, k is defined by (3-4ν) for plane stress conditions and (3-4ν)/(1+ν) for plane strain conditions, r and θ are the polar coordinates defined at the crack tip, and A_i are parameters depending on the geometry and load on the specimen.

The near tip nodal displacements at nodes b, c, d and e as shown in Fig. 1a, are of interest to calculate the SIFs under mixed mode conditions. The displacements are extrapolated by considering equation (1) along the crack faces $\theta = \pm\pi$. Particularizing for nodes b and c on the singular element at the upper face of the crack gives [23] :

$$v_b = K_I\sqrt{\frac{2}{\pi}}\frac{(1+v)(k+1)}{4E} + \sqrt{L} - \frac{A_2(1+v)(k+1)}{12E}L^{3/2} + O(L^{5/2}) \tag{2}$$

$$v_c = K_I\sqrt{\frac{2}{\pi}}\frac{(1+v)(k+1)}{2E} + \sqrt{L} - \frac{2A_2(1+v)(k+1)}{12E}L^{3/2} + O(L^{5/2}) \tag{3}$$

Where L, is the length of the element side connected to the crack tip.

Using equations (2) and (3), the SIF K_I can be obtained as:

$$K_I = \frac{E}{3\,(1+\upsilon)(1+k)}\sqrt{\frac{2\pi}{L}}\,(8v_b - v_d) \tag{4}$$

The nodal displacements at the other two nodes can be estimated by the similar technique. If one considers the all four nodal displacements, the SIF K_I can be expressed as:

$$K_I = \frac{E}{3\,(1+\upsilon)(1+k)}\sqrt{\frac{2\pi}{L}}\left[4(v_b - v_d) - \frac{(v_c - v_e)}{2}\right] \tag{5}$$

The displacement tangential to the crack plane, u under pure mode II loading, is given by:

$$u = K_{II}\frac{1+\upsilon}{4E}\sqrt{\frac{2r}{\pi}}\left[(2k+1)\,sin\frac{\theta}{2} - sin\frac{3\theta}{2}\right] + \frac{A_1(1+\upsilon)r}{E}(k-3)\,sin\,\theta + \frac{A_2(1+\upsilon)r^{\frac{3}{2}}}{E}\left[\frac{(2k-1)}{3}\sin\frac{3\theta}{2} - \sin\frac{\theta}{2}\right] + \cdots \tag{6}$$

Where K_{II} is the SIF under pure mode-II loading.

Similarly, using the nodal displacements of the four nodes, the SIF K_I can be expressed as:

$$K_{II} = \frac{E}{3\,(1+\upsilon)(1+k)}\sqrt{\frac{2\pi}{L}}\left[4(u_b - u_d) - \frac{(u_c - u_e)}{2}\right] \tag{7}$$

For isotropic FGM materials, elastics properties E and ν are evaluated at the crack-tip. The mixed mode SIFs K_I and K_{II} are given by [24-25]:

$$K_I = \frac{E_{tip}}{3\,(1+\upsilon_{tip})(1+k_{tip})}\sqrt{\frac{2\pi}{L}}\left[4(v_b - v_d) - \frac{(v_c - v_e)}{2}\right] \tag{8}$$

$$K_{II} = \frac{E_{tip}}{3\,(1+\upsilon_{tip})(1+k_{tip})}\sqrt{\frac{2\pi}{L}}\left[4(u_b - u_d) - \frac{(u_c - u_e)}{2}\right] \tag{9}$$

Fig.1a shown the special quarter point finite elements proposed by Barsoum [26]. These elements are used investigated to obtain a better approximation of the field near crack-tip; the mid-side node of the element in the crack-tip is moved to 1/4 of the length of the element, as shown in Fig.1b.

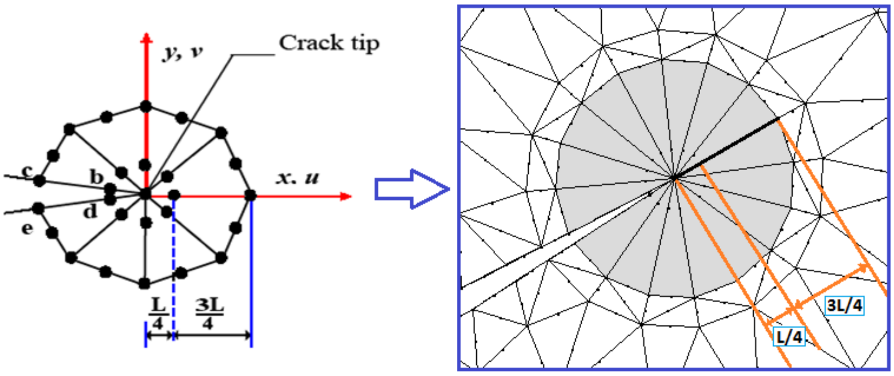

Fig. 1 – Special quarter point FE used for displacement technique

In this study, the Ansys Parametric Design Language has been investigated for creating the subroutine to simulate the fracture of isotropic FGMs, under mixed mode loadings. The displacement extrapolation method (DET) is employed, to determine the SIFs using Eqs. (8) and (9). The examples proposed in this paper are studied for different crack lengths. Each crack size consists of :

1. Input the geometrical model for pre-cracked FGM structure.

2. Input the variation continues of the elastics properties are implemented using Ansys Parametric Design Language code.
3. Meshing of the geometrical model by 2D isoparametric element: in this step, the 8-node isoparametric element (PLANE183) is employed for mesh generation in cracked FGM plate, as shown in Fig. 2(a). The main characteristic of 2D- PLANE183 element is the fact that the middle node of the element is translated on one quarter of the element side. This transformation changes the shape function to model the singularity at the crack tip (Fig. 2(b)).
4. FE analysis: Solve the problem as a static analysis.
5. Calculation of SIFs from Eqs. 9 and 10: The implementation of the present technique using APDL code is described in appendix A.

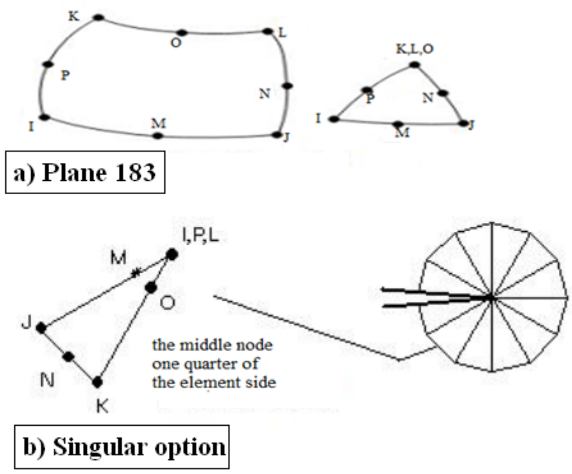

Fig. 2 – Singular elements for 2-D models using the element of PLANE183

3 Results and discussion

The performance of the displacement extrapolation technique to calculate the SIFs in isotropic FGMs is analysed by means of finite element method. For this purpose, two examples are investigated:

a) An isotropic FGM disk with a central crack subjected to concentrated loads.

b) A crack parallel to material gradation in three-point bending specimen.

3.1 FGM Disk with an inclined center crack

In this problem, consider a FGM disk with a central inclined crack of θ=30° subjected to concentrated forces. The geometry of FGM disk, boundary conditions and crack length are illustrated in Fig. 3a. The FGM disk is considered under plane stress conditions and the variation of Young's modulus E along the radial direction is expressed by [27]:

$$E(r) = \bar{E} e^{\beta r} \tag{10}$$

r is the disk radius, with $r = \sqrt{X_1^2 + X_2^2}$

X_1 and X_2 are Cartesian coordinates.

A point load P=± 100 units is applied to the top and bottom of the FGM disk, at coordinate nodes $(X_1, X_2) = (0, \pm 10)$. The displacements $u_1=0$ at the coordinate node $(X_1, X_2) = (-10, 0)$ and $u_2=0$ at the coordinate node $(X_1, X_2) = (\pm 10, 0)$.

The following data are investigated for the FEM study:

$\bar{E}$=1, ν=0.3, R=10, a=1, β= (0.5, 0.25, 0, -0.25, -0.5,),

The FGM disk is meshed by 8-node isoparametric element as shown in Fig. 3b. A special mesh is investigated around crack tips to consider the singularity of stress and deformations fields (Fig. 3c). This mesh will be employed to calculate the mixed mode SIFs, using eqs. 8 and 9.

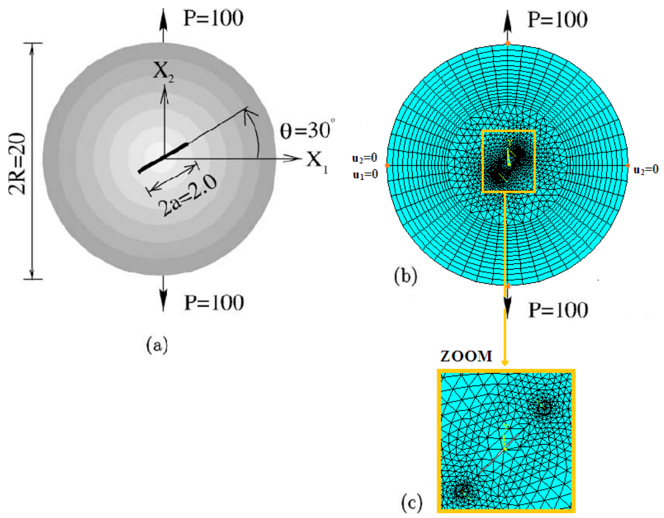

Fig. 3 – FGM Disk with an inclined center crack - a) Geometrical model with boundary conditions - (b) Typical mesh model - (c) Special finite elements around crack tips

Figs (4a) and (4b) show respectively, the variation of the SIFs K_I and K_{II} as a function of the parameter β (with a/R=0.1 and θ=30°).Using present method, the results obtained for K_I and K_{II} are compared with those reported by Kim and Paulino [12], by means of modified crack closure method (MMC) and interaction integral method (M-Integral). In addition, the SIFs are also calculated by the displacement correlation technique (DCT) adopted in reference [12]. This comparison indicates a good approximation between the four techniques.

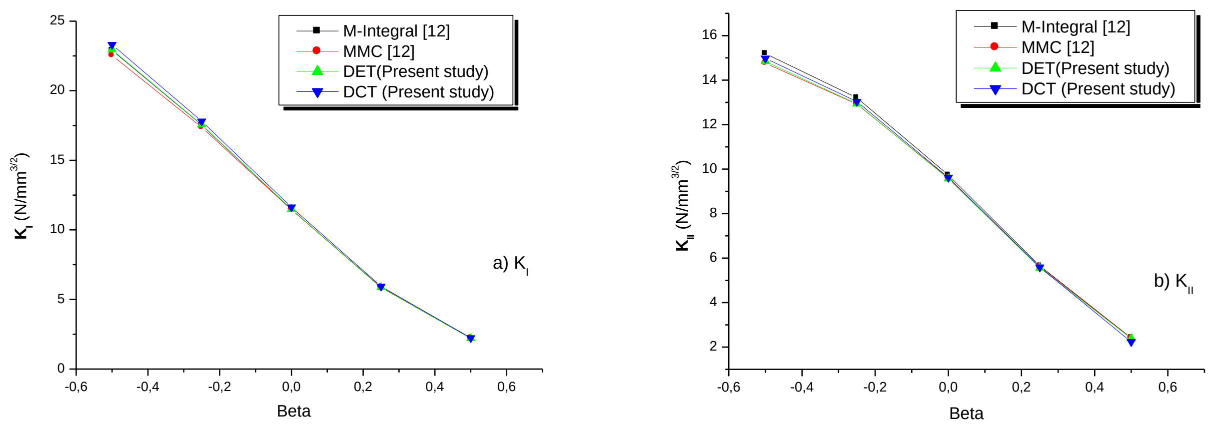

Fig. 4 – Variation of K_I and K_{II} vs. β - a) Variation of K_I - b) Variation of K_{II}

For reasons of symmetry in the model geometrical and the boundary conditions, the evolution of the SIFs K_I and K_{II} as a function of the angles of orientation θ is studied in the interval [90°, -90°]

Figs. (5) and (6) shows the variation of the SIFs K_I and K_{II} as a function of the positive and negative angles of crack direction, considering various size of initial crack (a/w) and β = -0.75. The obtained results from present technique allow us to deduce the following conclusions:

- The SIF K_I is maximum when the crack direction θ=0°, which is normal since it is in the presence of the opening mode, then it decreases progressively towards negative values with the increase the crack direction θ as shown in Fig. 5.
- The negative values of K_I obtained for orientations vary between [90°, 76°] and [-90°, -76°]. These orientations present a risk of propagation of the crack by shear mode.
- The SIF K_I increases with the crack length. The curves plotted for each crack size and for positive and negative crack directions are symmetrical to each other with respect to the ordinate axis (yy '), as shown in Fig. 5.
- In the case of positive angles (Fig. 6), the SIF K_{II} =0 when the crack direction θ=0° or well reached (θ=90°) and increases proportionally as a function of angle until at a maximum value corresponds to an angle θ=45° from which the curve takes a descending form with the increase of the crack direction θ. This evolution has been noticed for homogeneous materials case, considering numerical data reported in references [28] and [29].
- The K_{II} curves obtained for given crack length and for positive and negative crack directions are symmetric compared to the point M of coordinates (x, y) = (0,0) (Fig. 6). These results indicate that the evolution of K_{II} is dependent on the crack direction angle.

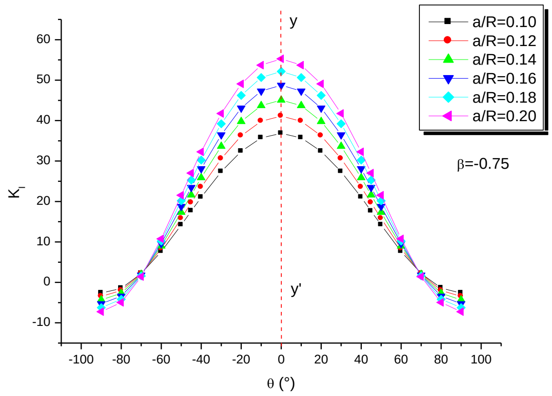

Fig. 5 – Variation of K_I vs. crack angles θ (β = -0.75)

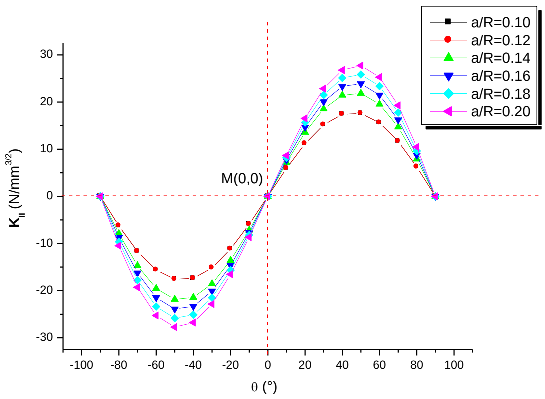

Fig. 6 – Variation of K_{II} vs. crack angles θ (β = -0.75)

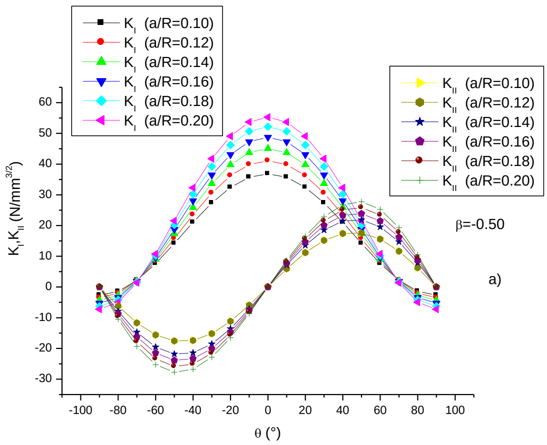

Fig. 7 – Variation of K_I and K_{II} vs. crack angles θ (β = -0.50)

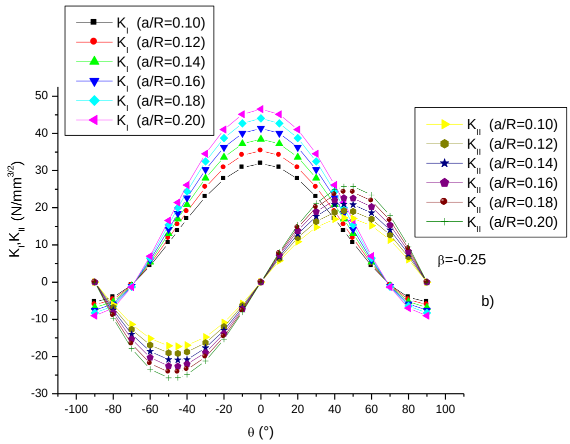

Fig. 8 – Variation of K_I and K_{II} vs. crack angles θ (β = -0.25)

Figs. (7 ~11) show the variation of the SIFs K_I and K_{II} as a function of the positive and negative angles of the crack, using different values of β (-0.50, -0.25, 0, 0.25, 0.50). The evolution of SIFs K_I and K_{II} are plotted in the same figure for each value of β.

The same variation of SIFs K_I and K_{II} is obtained as that observed for β = -0.50. The SIF K_I is maximum when the crack direction is zero, then it gradually decreases to negative values with the increase of the crack angle θ.

The range of negative values depending the parameter β, i.e. according to the variation of Young's modulus E along the radial direction.

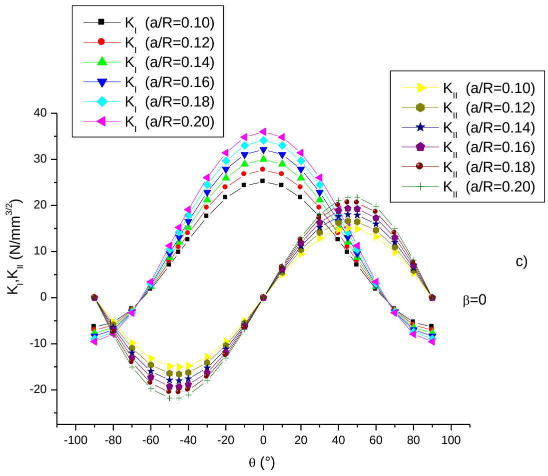

Fig. 9 – Variation of K_I and K_{II} vs. crack angles θ (β =0)

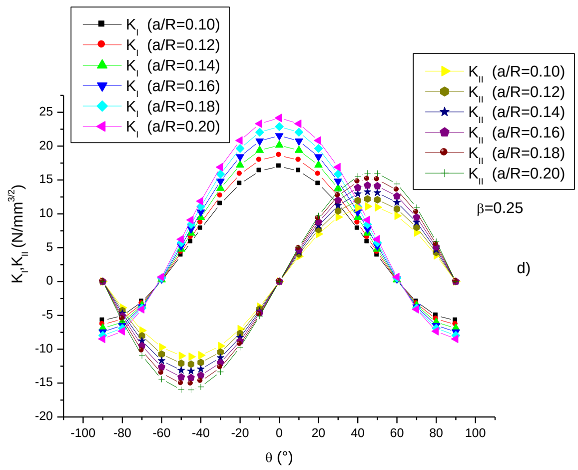

Fig. 10– Variation of K_I and K_{II} vs. crack angles θ (β = 0.25)

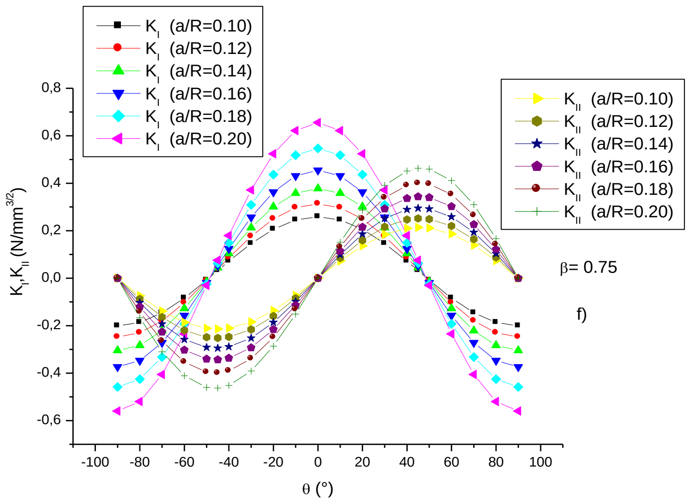

Fig. 11 – Variation of K_I and K_{II} vs. crack angles θ (β = 0.50)

3.2 FGM beam subjected to three-point bending

In this example, three-point bending specimen with crack parallel to material gradation is considered. Fig. 12 shows the three-point bending specimen geometry and boundary conditions. For this study, the Poisson's ratio ν is assumed to be constant and the variation of Young's modulus E is considered linear along the thickness direction, expressed by [11-12]:

$$E(x_2) = Ax_2 + B \tag{11}$$

with: $A = \frac{(E_2 - E_1)}{2h}$ and $B = \frac{(E_2 + E_1)}{2}$

The following data are investigated for the present study:

2H = 10 units, L = 54 units, t = 1 unit and force P = 1 unit is applied to the specimen center, under plane stress conditions.

The FGM beam is meshed with 8-node isoparametric element and in particular, special elements are employed to characterize the singularity around the crack tip, as shows in Fig.13. The SIFs K_I are evaluted for three crack sizes: a=4.5, 5 and 5.5 units.

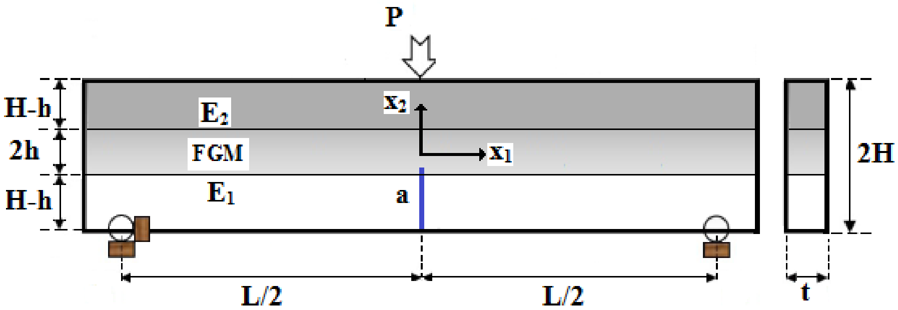

Fig. 12– FGM beam geometry with boundary conditions

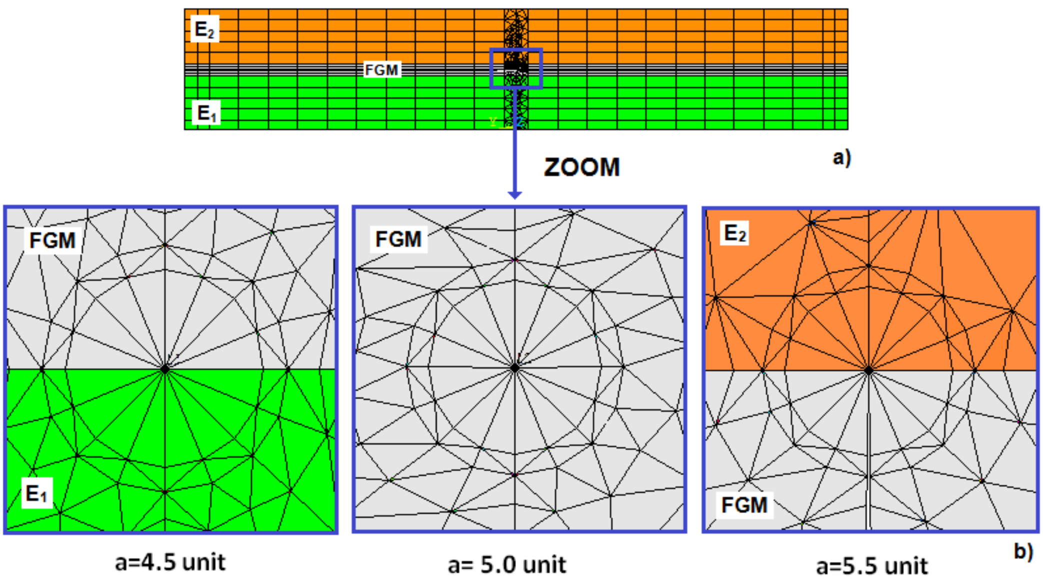

Fig. 13– Typical FE mesh of FGM beam: (a) Typical mesh model (b) Detail mesh around crack tip

For the three crack sizes, Fig.14 illustrated the evolution of the normalized SIF $(\frac{K_I\sqrt{H}}{P})$ according to the ratio E_1/E_2. The results obtained by FE method using present technique, are compared with obtained by Kim and Poulino [12], using MMC, J*-Integral and DCT methods.

The plotted curves show that the results obtained by DET are identical with those obtained by the three methods. The application of present technique is valid for other geometries, with other formulas for the material properties of FGMs.

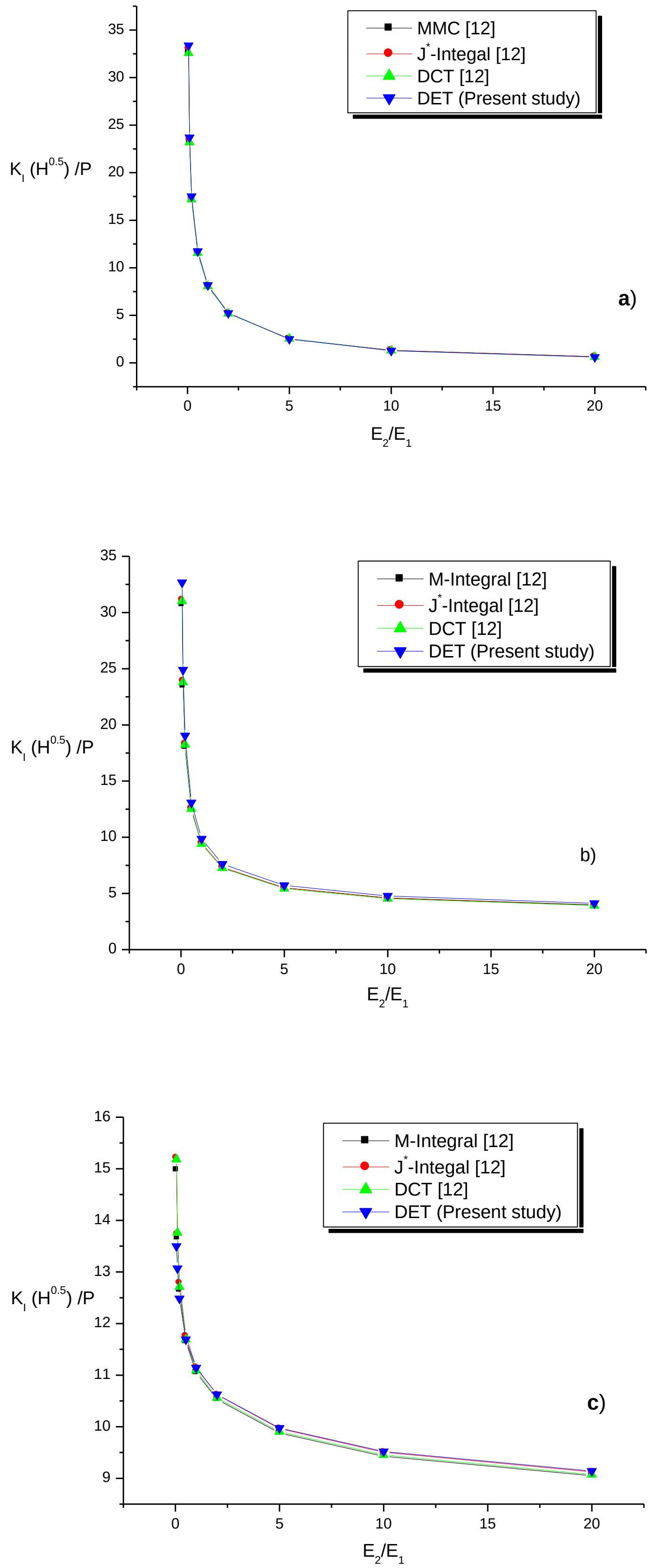

Fig. 14 – Variation of normalized SIF vs. E_1/E_2: a/2H= 0.45 - b) a/2H=0.50 - c) a/2H=0.55

4 Conclusion

In this paper, the mixed-mode stress intensity factors in functionally graded materials subject to quasi-static loadings are evaluated by means of an extrapolation displacement technique in conjunction with the 2D finite element analyses. The obtained results allow us to deduce the following conclusions:

– The singular elements proposed by Barsoum are investigated to consider the singularity of stress and deformations fields around the crack tip.

– The variations continue of the materials properties has been successfully incorporated in finite element code, using the variation of Young's modulus along the radial direction and the thickness direction, respectively.

– The extrapolation displacement technique proves to be an accurate and robust scheme in the numerical examples where various types of material gradation, exponential and linear are considered. The FEM results showed very good agreement with other published results.

– The material gradation and the crack direction may have a significant influence on SIFs under mixed mode loading.

Based on the FE numerical results of the present paper, it was recommended to add further development in the software to analysed the SIFs for orthotropic FGMs subjected to thermomechanics, dynamic or under other loading conditions. In addition, the simple formulation of present method also makes it possible to develop a methodology for solving the crack propagation problems under plane stress and plane strain conditions.

Appendix A. User subroutine

```
!-------------Extrapolation technique-----------------------------------
/POST1
*GET,xx2,kp,8,LOC,X
*GET,yy2,kp,8,LOC,y
*status,xx2
*status,yy2
LOCAL,11,0,xx2,yy2,0,alpha1, , ,1,1,
CSYS,11,
RSYS,11
!------------Extrapolation technique DET--------
*if,technique,eq,1,then
*GET,UX_9,NODE,9,U,X
*GET,UY_9,NODE,9,U,Y
 *GET,UX_7,NODE,7,U,X
*GET,UY_7,NODE,7,U,Y
  *GET,UX_10,NODE,10,U,X
*GET,UY_10,NODE,10,U,Y
  *GET,UX_6,NODE,6,U,X
*GET,UY_6,NODE,6,U,Y
E_tip=(E_all)*EXP(alpha*a)
 *SET,nuxy,0.3
*SET,EX,E_tip !
!*SET,HH,0.4  !H Rayon de la rossette
*SET,K_DP,3-(4*0.3)
*SET,K_CP,(3-0.3)/(1+0.3)
*SET,AA,EX/(3*(1+0.3)*(1+k_DP))
*SET,B,SQRT(6.28/HH)
*SET,C11,4*(UY_9-UY_7)
```

```
*SET,C22,4*(UX_9-UX_7)
*SET,D11,0.5*(UY_10-UY_6)
*SET,D22,0.5*(UX_10-UX_6)
*SET,KI, AA*B*(C11-D11)
*SET,KII,AA*B*(C22-D22)
!-------------Correlation technique DCT--------
E_tip=(E_all)*EXP(alpha*a)
*status,E_tip
!E_tip=1.0789
*GET,UX_b,NODE,9,U,X
*GET,UY_b,NODE,9,U,Y
 *GET,UX_d,NODE,7,U,X
*GET,UY_d,NODE,7,U,Y
  *GET,UX_c,NODE,10,U,X
*GET,UY_c,NODE,10,U,Y
  *GET,UX_e,NODE,6,U,X
*GET,UY_e,NODE,6,U,Y
 *SET,nuxy,0.3
!-------------------------------------
*SET,K_DP,3-(4*0.3)       !plane strain
*SET,K_CP,(3-0.3)/(1+0.3)  !plane stress
!-------------------------------------
*SET,AA,(E_tip)/(2*(1+0.3)*(1+k_DP))
*SET,BB,SQRT(6.28/HH)
*SET,C1,4*((UY_b)-(UY_d))  ! i-1
*SET,C2,4*((Ux_b)-(Ux_d))  ! i-1
*SET,D1,((UY_c)-(UY_e))  ! i-2
*SET,D2,((Ux_c)-(Ux_e))  ! i-2
*SET,KI, AA*BB*(C1-D1)
*SET,KII,AA*BB*(C2-D2)
!---------------------------------------------
```

REFERENCES

[1]- S.K. Chan, I.S. Tuba, W.K. Wilson, On the finite element method in linear fracture mechanics. Eng. Fract. Mech. 2(1970)1–17. doi:10.1016/0013-7944(70)90026-3

[2]- C.F. Shih, H.G. De Lorenzi, M.D. German, Crack extension modeling with singular quadratic isoparametric elements. Int. J. Fract. 12(1976) 647–51. doi:10.1007/BF00034654

[3]- D.M. Parks, A stiffness derivative finite element technique for determination of crack tip stress intensity factors. Int. J. Fract. 10(1974) 487–502. doi:10.1007/BF00155252

[4]- H.G. De Lorenzi, Energy release rate calculations by the finite element method. Eng. Fract. Mech. 21 (1985) 129–43. doi:10.1016/0013-7944(85)90060-8

[5]- B. Moran, C.F. Shih, A general treatment of crack tip contour integrals. Int. J. Fract. 35(1987) 295–310. doi:10.1007/BF00276359

[6]- P. Gu, M. Dao, R. Asaro, A simplified method for calculating the crack-tip field of functionally graded materials using the domain integral. J. Appl. Mech. 66(1999) 101–8. doi:10.1115/1.2789135

[7]- A. Anlas, M.H. Santare, J. Lambros, Numerical calculation of stress intensity factors in functionally graded materials. Int. J. Fract. 104(2) (2000) 131–143. doi:10.1023/A:1007652711735

[8]- J. Yau , S. Wang, H. Corten, A mixed-mode crack analysis of isotropic solids using conservation laws of elasticity. J. Appl. Mech. 47(2) (1980) 335–41. doi:10.1115/1.3153665

[9]- J. Dolbow, M. Gosz, On the computation of mixed-mode stress intensity factors in functionally graded materials.

Int. J. Solids. Struct. 39 (2002) 2557–74. doi:10.1016/S0020-7683(02)00114-2
[10]- X.W. Gao, Ch. Zhang, J. Sladek, V. Sladek, Fracture analysis of functionally graded materials by a BEM. Compos. Sci. Technol. 68 (2008) 1209–1215. doi:10.1016/j.compscitech.2007.08.029
[11]- B.N. Rao, S. Rahman, Mesh-free analysis of cracks in isotropic functionally graded materials. Eng. Fract. Mech. 70 (2003) 1–27. doi:10.1016/S0013-7944(02)00038-3
[12]- J.H. Kim, H. Glaucio, G.H. Paulino, Finite element evaluation of mixed mode stress intensity factors in functionally graded materials. Int. J. Numer. Methods. Eng. 53(8) (2002) 1903–1935. doi:10.1002/nme.364
[13]- S. Dag, B. Yildirim, Computation of Thermal Fracture Parameters for Inclined Cracks in Functionally Graded Materials Using Jk-Integral. J. Therm. Stresses. 32 (2009) 530–556. doi:10.1080/01495730802637480
[14]- B. Yildirim, S. Yilmaz, S. Kadioglu, Delamination of Compressively Stressed Orthotropic Functionally Graded Material Coatings under Thermal Loading. J. Appl. Mech, 75(5) (2008) 051106. doi:10.1115/1.2936239
[15]- I. Eshraghi, N. Soltani, Stress intensity factor calculation for internal circumferential cracks in functionally graded cylinders using the weight function approach. Eng. Fract. Mech. 134 (2015) 1–19. doi:10.1016/j.engfracmech.2014.12.007
[16]- N. Benamara, A. Boulenouar, M. Aminallah, Strain Energy Density Prediction of Mixed-Mode Crack Propagation in Functionally Graded Materials. Period. Polytech. Mech. Eng. 61(1) (2017) 60–67. doi:10.3311/PPme.9682
[17]- J.H. Kim, G.H. Paulino, Mixed-mode J-integral formulation and implementation using graded finite elements for fracture analysis of nonhomogeneous orthotropic materials. Mech. Mater. 35(1) (2003) 107–128. doi:10.1016/S0167-6636(02)00159-X
[18]- Y. Ait Ferhat, A. Boulenouar, N. Benamara, L. Benabou. Generalized displacement correlation method for mechanical and thermal fracture of FGM. Int. J. Comput. Mater. Sci. Surf. Eng. accepted 2020. doi:10.1142/S2047684120500049
[19]- ANSYS, Inc. Programmer's Manual for Mechanical APDL. Release 12.1, 2009.
[20]- A. Boulenouar, N. Benseddiq, M. Mazari, N. Benamara, FE model for linear elastic mixed mode loading: estimation of SIFs and crack propagation. J. Theor. Appl. Mech. 52(2) (2014) 373–383
[21]- A. Boulenouar, A. Benouis, N. Benseddiq, Numerical modelling of crack propagation in cement PMMA: Comparison of different criteria. Mat. Res. 19 (4) (2016) 846–855. doi:10.1590/1980-5373-MR-2015-0784
[22]- A. Benouis, A. Boulenouar, B. Serier, Finite element analysis of the behaviour of a crack in the orthopedic cement. J. Theor. Appl. Mech. 54(1) (2016) 277–284. doi:10.15632/jtam-pl.54.1.277
[23]- M. Souiyah, A. Muchtar, A.K. Ariffin, A. Malek, M.I. Fadhel, B. Abu Zneid, Finite Element Model of Crack Growth under Mixed Mode Loading. Int. J. Mater. Eng. 2(5) (2012) 67–74. doi:10.5923/j.ijme.20120205.02
[24]- N. Benamara, A. Boulenouar, M. Aminallah, N. Benseddiq, On the mixed-mode crack propagation in FGMs plates: comparison of different criteria. Struct. Eng. Mech. 615(3) (2017) 371–379. doi:10.12989/sem.2017.61.3.371
[25]- M. Chafi, A. Boulenouar, A Numerical Modelling of Mixed Mode Crack Initiation and Growth in Functionally Graded Materials. Mat. Res. 615(3) 2019. doi:10.1590/1980-5373-mr-2018-0701
[26]- R.S. Barsoum, On the use of isoparametric finite element in linear fracture mechanics. Int. J. Numer. Methods. Eng. 10 (1974) 25–37. doi:10.1002/nme.1620100103
[27]- J.H. Kim, H. Glaucio, G.H. Paulino, The interaction integral for fracture of orthotropic functionally graded materials: evaluation of stress intensity factors. Int. J. Solids. Struct. 40(15) (2003) 3967–4001. doi:10.1016/S0020-7683(03)00176-8
[28]- A. Boulenouar, N. Benseddiq, M. Mazari, Strain energy density prediction of crack propagation for 2D linear elastic materials. Theor. Appl. Frac. Mech. 67–68 (2013) 29–37. doi:10.1016/j.tafmec.2013.11.001
[29]- C. Ki-Hyun, Y. Won-Ho, Fracture mechanics analysis on the bonded repair of a skin/stiffener with an inclined central crack. Comp. Struc. 55(3) (2002) 269–276. doi:10.1016/S0263-8223(01)00163-5